An Optimal Time Path Model of Eliminating Bad Loans

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Abstract The author developed a continuous optimal control model showing the optimal time path of issuing special treasury bills to the state-owned banks in the size of the bad loans. These debts should later be paid by the government with its dividend earnings as a shareholder of the banks. The government can achieve threefold effect from this process. First, it can eliminate the bad loans rather than only cover them. Second, it can soak up liquidity and refrain the banks from making too much loans. Third, making banks take the responsibility of bad loans by themselves at present will root out the moral-hazard behaviors of the banks and make them perform prudently in the future.
Keywords Bad loans, Optimal control, Special treasury bills, Time path

1 Introduction

The year of 2006 was the tenth anniversary of the Asian financial turmoil in 1997. It was a year when China's outstanding loans amounted to a record high 22.5 trillion yuan, with new lending of 3.18 trillion yuan[1]. This coincidence should serve as a warning to the Chinese government, because it was just the collapse of the careless bank sectors that sparked the currency crisis in those countries [2].

In reviewing the lessons of 1997 Asian financial turmoil, however, scholars found that governments should be at least partially blamed for the crisis. Although governments of the East Asian Countries did not take a continuous profligate fiscal policy as he first-generation models depicted[3][4], their guarantees to the banks in case of failings encouraged moral-hazard behaviors of the bank sector[5]. When the bubbles created by over-lending were broken, crisis was inevitable.

Chinese government did guarantee the banks. It wiped out the 3 trillion yuan of bad loans burdening its bank sector by cleaning them from the banks' books to that of the five Asset Management Companies sponsored by the Ministry of Finance and the People's Bank of China[6]. This encouraged the banks to perform carelessly in the future since bad loan could always be wiped out by the government. Nowadays, China's state-owned banks are indeed performing carelessly supported by the ample liquidity from trade surplus.

The People's Bank of China uses reserve requirements, interest rates and commercial bills sales to restrain banks' ability to make loans [7]. The result of this policy mix, however, is quite poor. New lending was 981 billion yuan for the first two months of 2007 combined, almost a third of the total for all of 2006 [8]. The central government seems like an unwise parent, who requires his children banks not to make imprudent new loans on one hand, and forgive them of their imprudent loans in the past on the other. In such a game, the central government will surely be a loser. That's one of the reasons why the government can't curb the excessive investments which could end in asset bubbles.

With ample liquidity and disobedient behavior of the banks at present, it's time for the central government to ask the banks to pay for the bad loans by themselves. The author suggests issuing special treasury bills to the banks the size of their previous bad loans. These treasury bills are special in the sense that they could only be paid by the dividend allocated to the government as a shareholder of the banks.

In Section 2 of this paper, the author developed a continuous optimal control model showing the optimal time path of issuing the special treasury bills. Section 3 analyzes a version of the model calibrated to Chinese data. Section 4 is the conclusion.

2 The Model

By moving the bad loans of the four state-owned banks to those of the Asset Management

Companies who were sponsored by the Ministry of Finance and the People's Bank of China, the bad loans were actually burdened on the government [9]. Thus, banks enjoyed a free feast which would probably be paid by the whole population in the future. This is quite unfair. Issuing some special treasury bills to the banks and later paying the principal and interests accrued by the government with its dividend revenue as an owner of the banks implies that the banks themselves, not all taxpayers, will pay for the bad loans.

Although it can also help the government curb investment, the issuance of the special treasury bills in such a large amount can't be done at a dash. It must be a gradual process in accordance with the overall economic situation and government fiscal position. Since individual households are the ultimate holders of them, the optimal time path of issuing treasury bills depends on their decisions. With this in mind, let's first look at the maximum utility behavior of a representative household.

2.1 The Representative Household

We assume a continuous-time, perfect-foresight endowment economy populated by an infinitely lived representative household [10]. The representative agent maximizes inter-temporal utility [11]. We use the CES utility function to depict the relationship between consumption and utility. Then the inter-temporal utility of a representative household can be defined as:

$$U = \int_o^T \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \tag{1}$$

Here c_t is consumption, σ is the inverse of the elasticity of inter-temporal substitution. ρ is the discount factor. The household can borrow and lend at an constant interest rate of r. We assume that $r = \rho$ [12]. The household is subject to flow budget constrains given by:

$$\dot{b}_t + \dot{d}_t = y_t + r(b_t + d_t) + v_t - \left(c_t + \tau_t + \pi_t y_y + \dot{y}_t \right) \quad \text{for} \quad t \geq 0 \tag{2}$$

Throughout the paper, $\dot{x}_t$ denotes dx/dt. And all variables are measured as proportion of real output of the time in local currency. We assume the net foreign assets held by the house hold, d_t is zero, which is a reasonable assumption because of China's control on the capital account [13]. v_t is the transfer payment he receives and τ_t is the tax he pays. π_t denotes the inflation rate and y_t is the GDP at time t. $\pi_t y_t + \dot{y}_t$ is equal to the seigniorage revenue, $\dot{M}_t / p_t$ [14]. With these assumptions, the flow budget constraint can be reduced to:

$$\dot{b}_t = (y_t + r b_t + v_t) - \left(c_t + \tau_t + \pi_t y_t + \dot{y}_t \right) \quad \text{for} \quad t \geq 0 \tag{3}$$

On the right side of (3), the first parenthesis denotes the total income of the household; the second one shows the deductions. The difference between the two items is the amount that can be used in buying domestic debts.

The representative household maximizes (1) subject to (3) by choice of time paths for c_t and b_t subject to known paths for π_t, y_t, v_t and τ_t.

2.2 The Period of Issuing the Treasury Bills

The issuance of special domestic debts guaranteed by bank profits implies an increase of b_t, whose optimal time path is determined by the household above-said. We believe the government, as the owner of the banks, should allocate a proportion of the transfer payments earmarked to pay for the bad loans at the same time of issuing. Thus, the total increases in b_t as well as those in earmarked transfer payments during a certain period of time should be equal to the sum of bad loans. We denote the nonearmarked transfer payments as $\tilde{v}_t$, as $\tilde{v}_t = v_0 e^{\tilde{\gamma} \cdot t}$. This idea can be formulated as follows:

$$\varphi = \int_0^T (\dot{b}_t - \dot{f}_t)e^{-rt}dt + \int_0^T (\tau_t - \tilde{v}_t - g_t)e^{-rt}dt - \int_0^T (rb_t - rf_t)e^{-rt}dt + \int_0^T (\pi_t y_t + \dot{y}_t)e^{-rt}dt \quad (4)$$

Here φ denotes the amount of the bad loans to be financed by earmarked transfer payment and special debts. g_t is the government expenditure including government investments. f_t is the net foreign assets held by the government dominated in local currency. $(b_t - f_t)$ denotes the government's net liabilities. Since China's central bank governor Zhou Xiao-chuan said on March 20, 2007 that China will stop stockpiling its massive foreign exchange reserves[15], we can assume that $\dot{f}_t = 0$, so that $f_t = f_0$. Then, the formula turns to:

$$\varphi = \int_0^T (\dot{b}_t - rb_t)e^{-rt}dt + \int_0^T (\tau_t - \tilde{v}_t - g_t)e^{-rt}dt + \int_0^T rf_0 e^{-rt}dt + \int_0^T (\pi_t y_t + \dot{y}_t)e^{-rt}dt \quad (5)$$

By setting φ, we can calculate the time needed to disclose the bad loans with known time paths of y_t, v_t, τ_t, g_t, π_t and optimal time path of b_t.

2.3 The Optimal Time Path of Issuing the Treasury Bills

The question of a representative household's utility maximizing behavior subject to a flow budget constraint can be shown as such an optimal control problem:

$$MaxU = \int_o^T \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho t}dt \quad (1)$$

$$\text{s.t. } \dot{b}_t = (y_t + rb_t + v_t) - \left(c_t + \tau_t + \pi_t y_t + \dot{y}_t\right) \qquad t \geq 0 \quad (3)$$

$$b_0 = A \ (A \text{ is given}) \quad (6)$$

$$b_T = B \quad (7)$$

$$B \geq 0$$

Time T is the result of (5). We set the time paths of $y_t, g_t, v_t, \tau_t, \pi_t$ as: $\pi_t = \pi$; $y_t = y_0 e^{nt}$, with n as the economic growth rate; $g_t = g_0 e^{\alpha t}$, with α as the growth rate of government expenditure; $\tau_t = \tau_0 e^{\beta t}$, with β as the growth rate of tax revenue; $v_t = v_0 e^{\gamma t}$, with γ as the growth rate of government transfer payment. The fact that only a small part of the transfer payment can be earmarked for paying for the bad loans means $\gamma \geq \tilde{\gamma}$.

By writing a current-valued Hamiltonian on this problem, we get the first-order condition:

$$\frac{\partial H_c}{\partial c_t} = c_t^{-\sigma} - \lambda_t = 0$$

$$c_t^{-\sigma} = \lambda_t \quad (8)$$

λ_t is the current-valued Lagrange multiplier. λ_t, which is also the co-state variable, satisfies such a differential equation as:

$$\lambda_t = -\frac{\partial H_c}{\partial b_t} + r\lambda_t = r\lambda_t + r\lambda_t = 0 \quad (9)$$

(9) tells that λ_t does not change over time, which implies that consumption is constant too:

$$c = \lambda^{-\frac{1}{\sigma}} \quad (10)$$

From (10), we can infer that:

$$c_t = c_o \tag{11}$$

From the time 0 government budget constraint:

$$f_0 - b_0 = r(f_0 - b_0) + \tau_0 + \pi_0 y_0 + \dot{y}_0 - g_0 - v_0 \tag{12}$$

and time 0 household budget constraint:

$$\dot{b}_0 = (y_0 + rb_0 + v_0) - \left(c_0 + \tau_0 + \pi_0 y_0 + \dot{y}_0 \right) \tag{13}$$

We can get time 0 consumption:

$$c_0 = y_0 - (f_0 - rf_0) - g_0 \tag{14}$$

With the optimal time path of control variable c and the time paths of other variables above-set, we can have the differential equation of the state variable b_t:

$$\dot{b}_t - rb_t = y_t - c - g_t - (\pi y_t + \dot{y}_t) \tag{15}$$

The solution of this differential equation is:

$$b_t = e^{rt}(K + \int (y_t - c - g_t - \pi y_t - \dot{y}_t) e^{-rt} dt) \tag{16}$$

We can solve constant *K* by formula (6), and with time *T* decided by (5), we can get the ultimate value of domestic debt $b_T = B$. We assume that the government keeps fiscal balance during this period, so the increase from *A* to *B* is totally the special national debts guaranteed by later bank profits, with no increase in other domestic debts.

Tab. 1 Parameter Value

Parameter	Value	Description
y_0	1	Real Output
n	0.08	Growth rate of real Output (growth target published by the government)
r	0.384	Real interest rate (Nominal interest rate minus inflation rate)
b_0	0.2	Time 0 government non-indexed domestic debt
τ_0	0.2	Time 0 tax revenue
β	0.16	Growth rate of tax revenue (2 times of output growth rate in recent years)
g_0	0.17	Time 0 Government expenditure (including government investment)
α	0.16	Growth rate of government expenditure (set to equal to that of revenue)
c	0.4454	Time 0 consumption (calculated)
v_0	0.03	Time 0 transfer payment
γ	0.16	Growth rate of the overall transfer payment (set to equal to that of revenue)
$\tilde{\gamma}$	0.14	Supposed Growth rate of nonearmarked transfer payment
π	0.03	Inflation rate (the cap rate published by the People's Bank of China)
φ	0.2	Contingent liability to be financed
f_0	0.4	Time 0 government assets (suppose totally as foreign exchange reserve)

3 Properties of the Model

3.1 Model Calibration

Tab.1 shows the parameter values of our model. We take the end of year 2006 as time 0. The economic growth rate is set at 8%, which is the target level published by the government at the beginning of 2007[16]. China's government revenue has been growing at a rate of 20% at least for the past 5 year [17]. That's almost as high as twice that of GDP. So it is feasible to set the growth rate of tax revenue β, which is the majority of the government revenue, as 2 times of economic growth rate n. We assume that the government would try to keep fiscal balance during the period, that's why we set the sum of government expenditure and transfer payment equal to tax revenue. And also, to keep balance, the growth rate of government expenditure α and the growth rate of transfer payment γ match that of the tax revenue β. At the beginning of 2007, the central bank of China also set the cap level of inflation at 3% [18]. After the People's Bank of China promised to keep on the tight monetary policy, we can reasonably predict that the interest rate will be raised twice in year 2007. At a rate of 27 basis points each time in history, the interest rate could be raised to 6.84%. And the real interest rate would be 3.84%. About the amount of bad loans φ, we adopt the value estimated by scholars rather than those published by the government [19]. Consumption at time 0 was calculated according to formula (14).

Data at the end of 2006 was collected from the databank of the Ministry of Finance and the People's Bank of China.

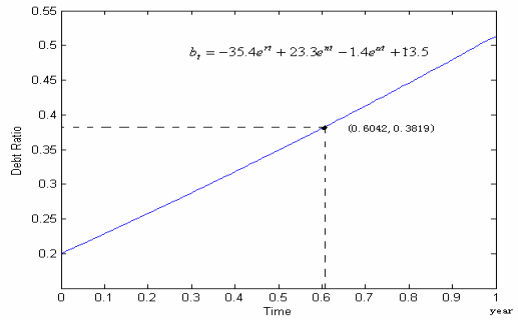

Fig.1 Optimal Time Path of Debt Ratio

3.2 Results

Fig. 1 depicts the optimal time path of domestic debts b_t. It shows that the government can issue domestic debts in the size of the bad loans in less than one year, 0.6 years of 8 months, with debt ratio reaching a quite sustainable 38.19%[20]. The time path is shows that an average 260 billion yuan of liquidity would be removed from the financial system every month. This can act against the monthly trade surplus and ease the pressure on Renminbi effectively.

3.3 Sensitivity Analysis

Fig. 2 (a) and (b) is the sensitivity analysis of debt ratio and time period on the growth rate of government expenditure α. The author assumes that α varies from 8% to 16%, while holding the growth rate of transfer payment and tax revenue at the benchmark value of 16%. It shows that both the debt ratio and time period is not sensitive to it.

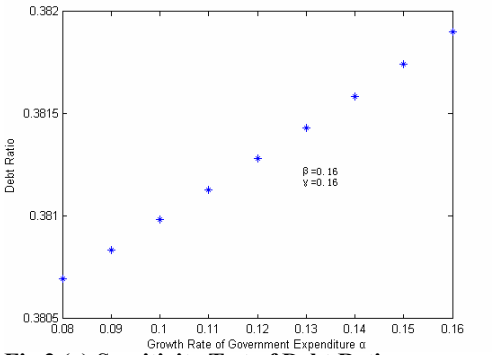

Fig 2 (a) Sensitivity Test of Debt Ratio on α

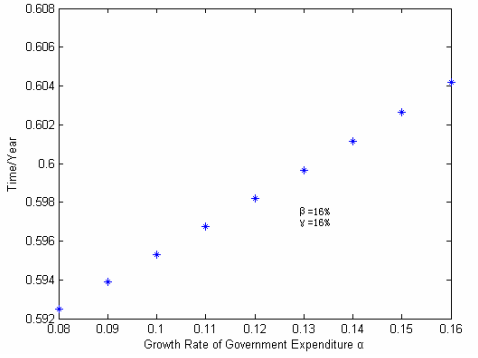

Fig 2 (b) Sensitivity Test of Time Period on α

Fig. 3 (a) and (b) is the sensitivity analysis of debt ratio and time period on the growth rate of

government transfer payment γ. The author assumes that γ varies from 8% to 16%, while holding the growth rate of government expenditure and tax revenue at the benchmark value 16%. It shows that both the debt ratio and time period is quite sensitive to it compared to α.

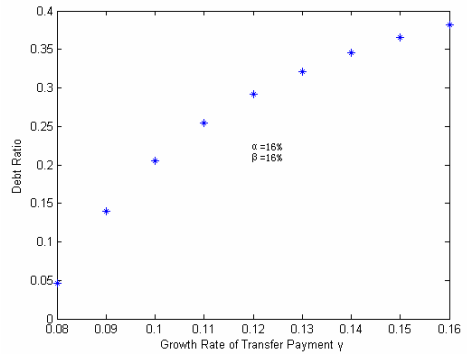

Fig 3 (a) Sensitivity Test of Debt Ratio on γ

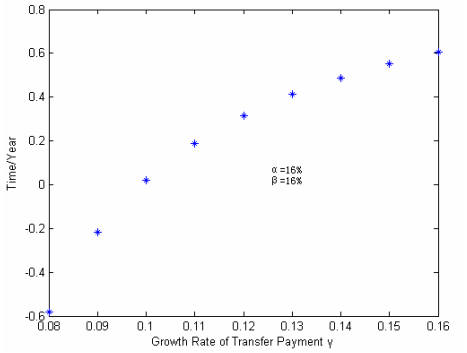

Fig 3 (b) Sensitivity Test of Time Period on γ

Fig.4 (a) and (b) is the sensitivity analysis on the growth rate of government revenue β. The author assumes that β varies from 8% to 16%, while holding the growth rate of transfer payment and government expenditure equal to it. It shows that debt ratio is more sensitive to the growth rate of government expenditure and transfer payment. It means that high speed of government revenue growth is essential to the elimination of bad debts.

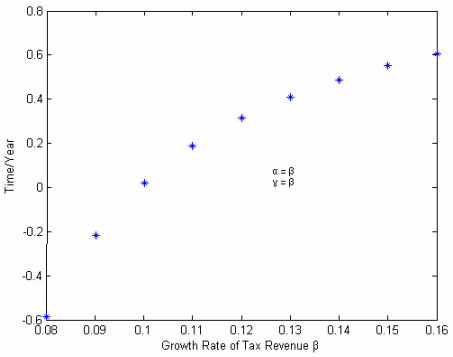

Fig 4 (a) Sensitivity Test of Debt Ratio on β

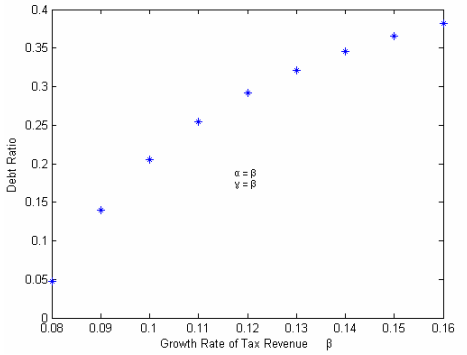

Fig 4 (b) Sensitivity Test of Time Period on β

4 Conclusion

Issuing special treasury bills guaranteed by later bank profits to the banks has threefold result. First, it can first disclose the bad loans then eliminate them gradually. Covered bad loans are contingent liabilities which would turn into realistic liabilities when fundamentals go bad.

Perspective deficits have been blamed for igniting 1997 Asian currency crisis [21]. So, the earlier the revealing of the contingent liabilities, the better. Second, it can soak up liquidity effectively so as to refrain the banks from making too much loans. Third, which is important in the long run, making banks take the responsibility of bad loans by themselves at present marks a turning point of the government's policy. The government may not bail out failing banks in the future. This will root out the moral-hazard behaviors of the banks and make them lend prudently in the future.

From the model, the author also shows that the idea is feasible given China's fundamentals. Fast economic growth and even faster tax revenue growth can help to eliminate bad loans in a short period and hold the debt ratio at a sustainable level at the same time.

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